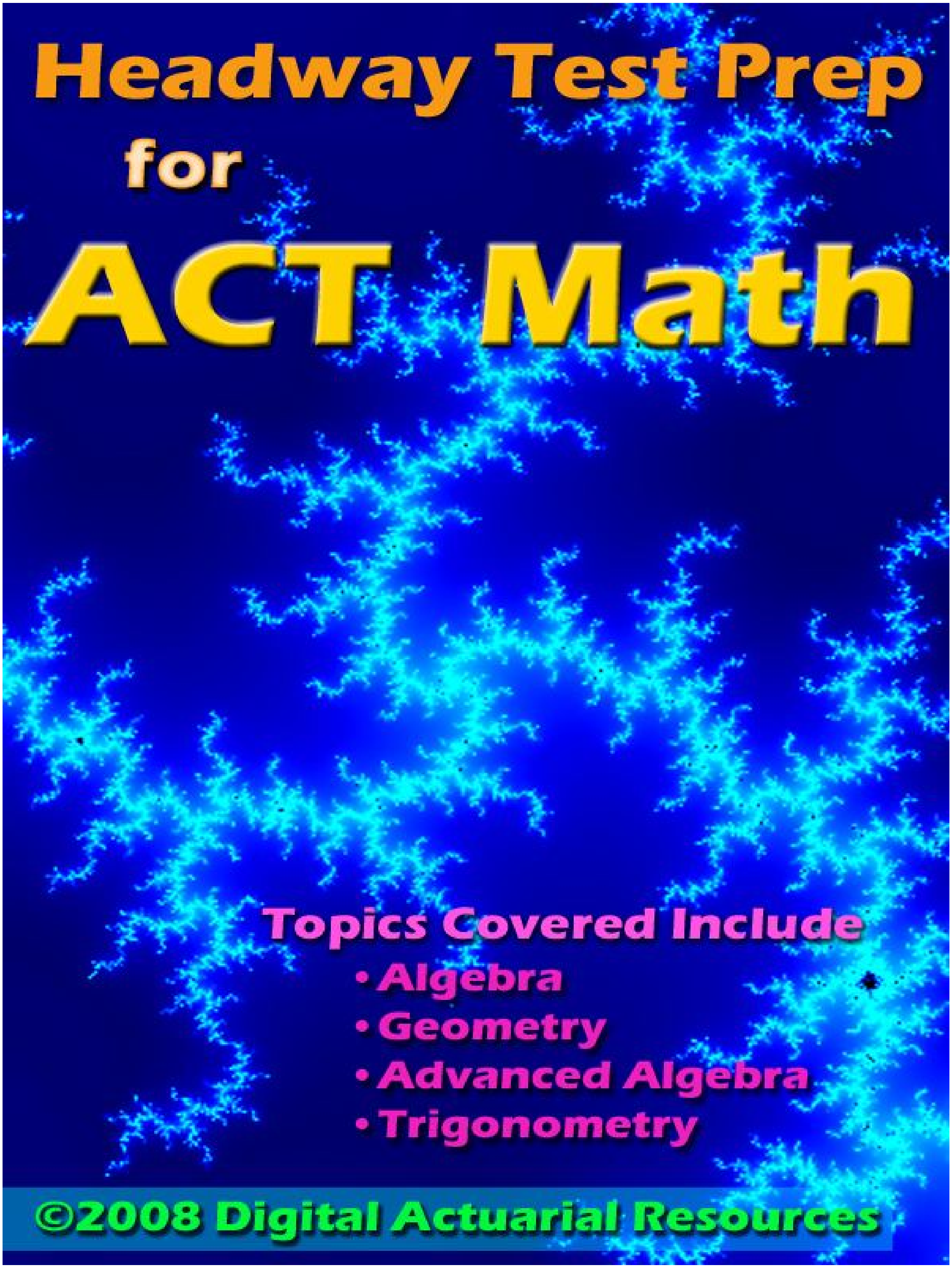
Headway Test Prep
for
ACT Math
Topics Covered Include
• Algebra
• Geometry
• Advanced Algebra
• Trigonometry
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Table of Contents

Section 10-4 Triangles

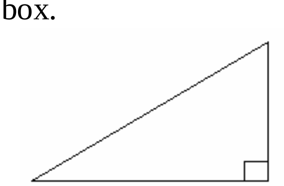
A *triangle* is a three-sided polygon with internal angles totaling $180°$. The triangle is the simplest polygon possible; you need at least three edges to form a closed polygon. Any side could be labeled the "base."

Triangles have several unique properties. First, no edge can be longer than the sum of the remaining two edges. When two angles are different, the opposite sides must be different. Correspondingly, when two edges differ, the opposite angles must be unequal.

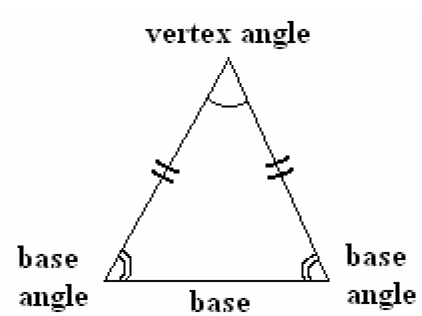
Types of Triangles and Angles

A triangle has three internal angles and three corresponding sides. The symbol for a triangle is a miniature triangle, Δ. The expression ΔABC reads "triangle ABC," where A, B, and C are the vertices. The angles within the triangle must add up to 180°. The size of an angle affects the length of the edge opposite that angle. The largest angle will be opposite from the largest side, and the smallest angle will correspond with the shortest edge.

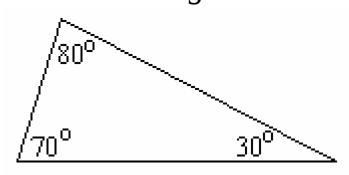
A *right triangle* has one angle that is 90°. The 90-degree angle is marked with a small box.



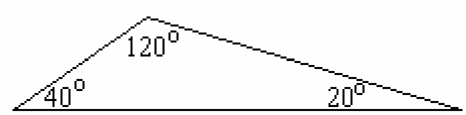
An *isosceles triangle* has two angles of the same degree. The two sides opposite the angles will also be the same length. Hash marks along the sides are used to illustrate that two edges have the identical length. Angles use small curves to show that two angles are of the same degree. Isosceles triangles have several unique terms. The two equal angles are called *base angles*. The edge adjacent to the two base angles is referred to as the *base*. The *vertex angle* is between the two equal edges.



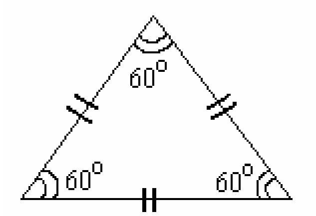
An *acute triangle* has three angles that are all less than 90°:



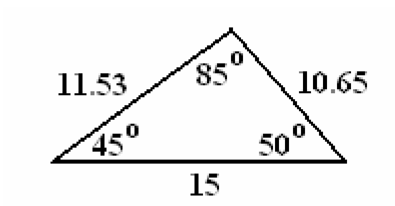
An *obtuse triangle* has one angle that is over 90°:



An *equilateral triangle* has three sides that are equal and three angles that are all the same. Every angle is exactly 60 degrees.

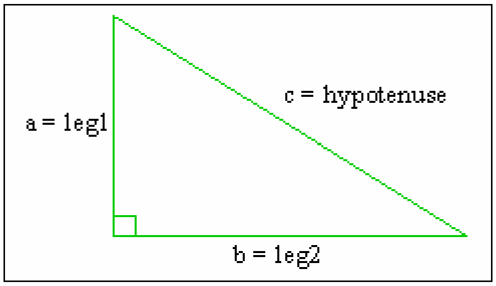


A *scalene triangle* has three completely unique side lengths and three different angles:



Solving Unknown Sides in a Right Triangle

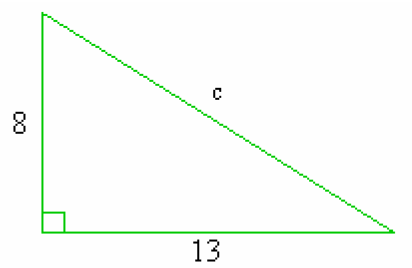
The *Pythagorean Theorem* states that given a right triangle, the square of the hypotenuse equals the sum of the squares of the two legs. Using the picture below, $a^2 + b^2 = c^2$.



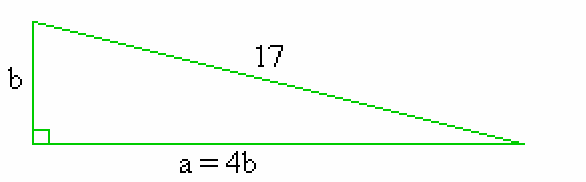
In a triangle, an interior angle is proportional to the size of the leg opposite the angle. The largest side is opposite the largest angle, and the smallest side is opposite the shortest angle.

Example
Find the missing side in each triangle below:

a.



b.



Solutions:

a. $a^2 + b^2 = c^2$

$8^2 + 13^2 = c^2$

$c^2 = 233 \quad \rightarrow \quad c = \sqrt{233} \approx 15.26434$

b. $a^2 + b^2 = c^2$

$(4b)^2 + b^2 = 17^2$

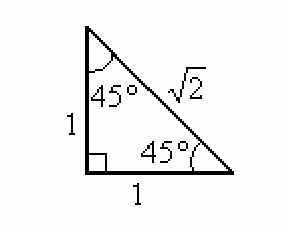
$16b^2 + b^2 = 17^2$

$17b^2 = 17^2$

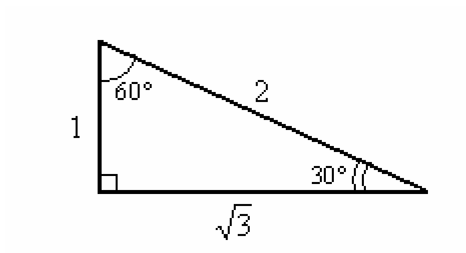
$b^2 = 17 \quad \rightarrow \quad b = \sqrt{17} \approx 4.12311 \quad \rightarrow \quad a = 4b \approx 16.49242$

Special Triangles

The 45-45-90 triangle is an isosceles/right triangle with two angles measuring $45°$. The diagram below also shows the proportions between the edges. If the legs meeting at the right angle are both of size 1, then the hypotenuse must have size $\sqrt{2}$. As an example, suppose one of the legs is 50 cm. The other leg must also be 50 cm., and the hypotenuse should measure $50*\sqrt{2} \approx 70.7107$ cm.

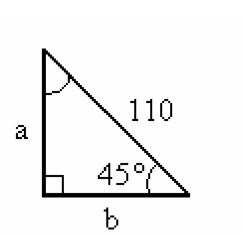


The 30-60-90 triangle is a right triangle with the listed angle measurements. This triangle also has shortcuts for computing missing sides. The figure below lists the proportions between the three edges. The edge opposite $30°$ has value 1, the side opposite $60°$ has length $\sqrt{3} \approx 1.7321$, and the edge opposite $90°$ is the longest with proportion 2. Suppose we know that the side opposite $30°$ measures 75 cm. The side opposite $60°$ must then have length $75*\sqrt{3} \approx 129.9038$ cm., and the edge labeled '2' measures $2*75$ = 150 cm.

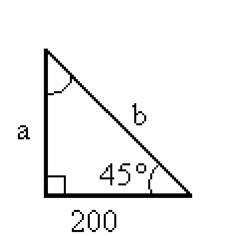


Example
Find the missing sides in these triangles:

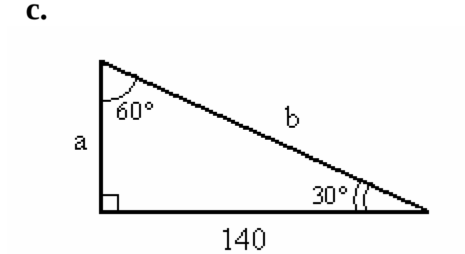
a.



b.



c.



Solutions:

a. $110 = \sqrt{2} * a \quad \rightarrow \quad a \approx 77.7817 \approx b$ (Since a = b)

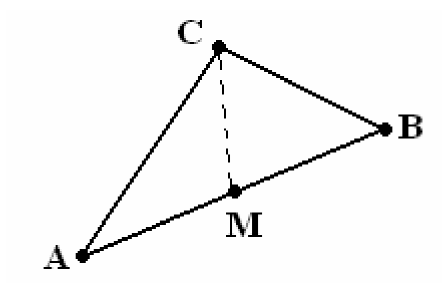
b. a = 200

$b = \sqrt{2} * 200 \approx 282.8427$

c. $140 = \sqrt{3} * a \quad \rightarrow \quad a \approx 80.829$
$b = a * 2 \approx 80.829 * 2 \approx 161.6581$

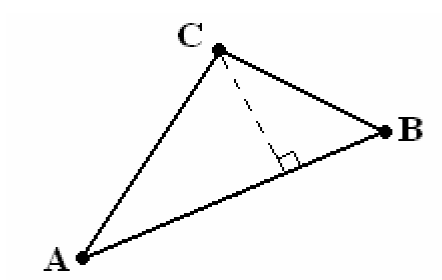
Median of Triangles

A *median* for a triangle connects a vertex with the midpoint on the opposing edge. The median is not necessarily perpendicular to the edge. Every triangle has three medians, with one drawn from each vertex.

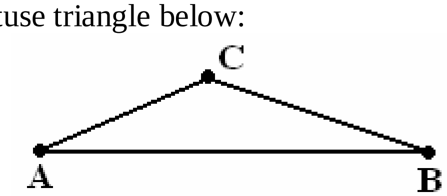


Height of Triangles

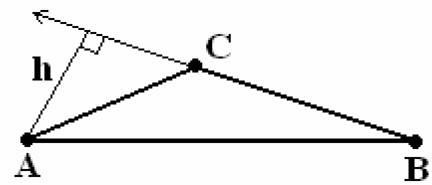
The *height* of a triangle is the length of a segment starting at any vertex and ending perpendicular to the opposite edge. The "opposite edge" is called the *base* of the triangle. Just as in the case of medians, a triangle has three possible height measurements.



You may need to extend the base in some triangles in order to find the height. Consider the obtuse triangle below:



Suppose we need to find the height as measured from vertex A. We must first link a segment from A to $\overline{CB}$, with the new segment being perpendicular to $\overline{CB}$. If we attempt to keep this new segment within the triangle, we find that no such segment is perpendicular to $\overline{CB}$. We need to artificially extend $\overline{CB}$ so that the height is perpendicular to it:



Area of a Triangle

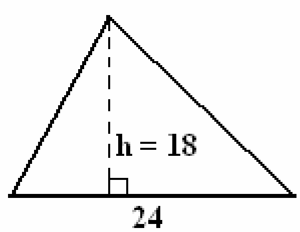
$Area = \frac{1}{2}$ * base * height

$A = \frac{1}{2}bh$

Example 1.1

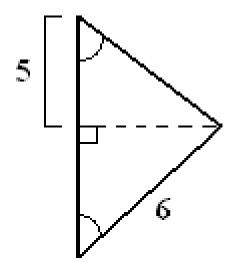
Find the areas of these triangles:

a.



b.

c.

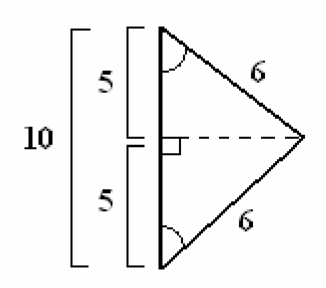


Solutions:

a. $A = \frac{1}{2} * 24 * 18 = 216$

b. $A = \frac{1}{2} * 13 * 9 = 58.5$

c. Using the fact that the figure is an isosceles triangle with two equal angles, we can derive the following attributes for the triangle:



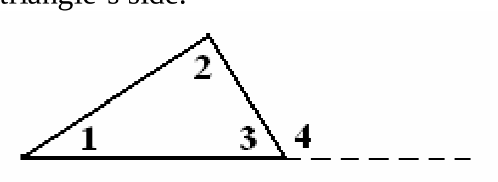
Use Pythagorean's Theorem to find the height.

$$6^2 = 5^2 + h^2$$
$$36 = 25 + h^2$$
$$11 = h^2$$
$$h = \sqrt{11} \approx 3.3166$$

$$A = \frac{1}{2} * 10 * 3.3166 \approx 16.5831$$

Exterior Angles for Triangles

The facts that a triangle has $180°$ of internal angles and supplementary angles add to $180°$ lead to a theorem about the external angles around a triangle. An external angle is formed by extending one of the triangle's sides. The external angle lies between the triangle's nearest actual edge and the extended edge. In the diagram below, we extend the bottom edge to the right. A new angle, $\angle 4$, is created between the extension and the nearby triangle's side.



Angles 3 and 4 are supplementary, summing to $180°$. Additionally, angles 1, 2, and 3 must total $180°$. To summarize,

$$\angle 1 + \angle 2 + \angle 3 = 180°$$
$$\angle 4 + \angle 3 = 180°$$

From the equations above, you can see that $\angle 1 + \angle 2 = \angle 4$.

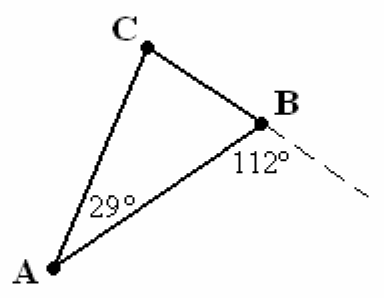
Triangle Exterior Angle Theorem

Given a triangle with three internal angles, $\angle 1$, $\angle 2$, and $\angle 3$, and an external angle $\angle 4$ which is supplementary to $\angle 3$, the following equation is true: $\angle 1 + \angle 2 = \angle 4$.

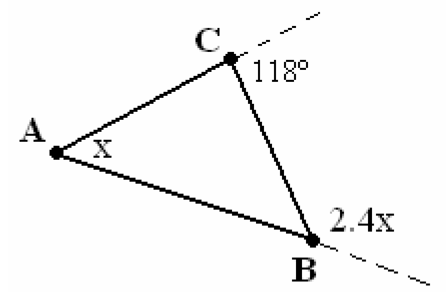
The Triangle Exterior Angle Theorem states that any external angle to a triangle has the same measurement as the remaining two, opposite angles inside the triangle.

Example
Use the Triangle Exterior Angle Theorem to solve for the missing internal angles:

a.



b.



Solutions:

a. $112° = \angle A + \angle C \rightarrow 112° = 29° + \angle C \rightarrow \angle C = 83°$

$180° = 112° + \angle B \rightarrow \angle B = 68°$

b. $180° = 118° + \angle C \rightarrow \angle C = 62°$

$118° = \angle A + \angle B$

$118° = x + (180° - 2.4x)$

$118° = x + 180° - 2.4x$

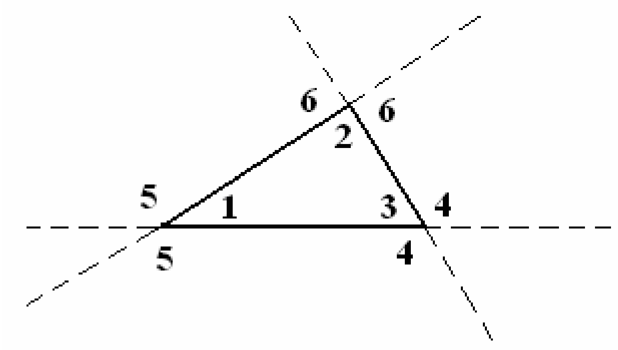
$118° = 180° - 1.4x$

$-62° = -1.4x \rightarrow x = 44.2857° = \angle A$

$180° = \angle A + \angle B + \angle C$

$180° = 44.2857° + \angle B + 62° \rightarrow \angle B = 73.7143°$

It is possible to extend all the sides around the triangle, creating six exterior angles. The extended sides form vertical angles at each vertex, which are equivalent to each other.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6
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